

QUANTUM MEASUREMENT PROBLEM
AND THE POSSIBLE ROLE OF THE GRAVITATIONAL FIELD

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Abstract

The quantum measurement problem and various unsuccessful attempts to resolve it are reviewed. A suggestion by Diosi and Penrose for the half life of the quantum superposition of two Newtonian gravitational fields is generalized to an arbitrary quantum superposition of relativistic, but weak, gravitational fields. The nature of the “collapse” process of the wave function is examined.

1 Introduction

Two of the most important unsolved problems in theoretical physics are the problem of quantizing gravitation and the measurement problem in quantum theory. It is possible that the solution of each one needs the other. Since we have successful quantum theories of electroweak, and strong interactions, the solution to the problem of the collapse of the wave function, known as the mea-

surement problem, may lie in the yet unknown quantum theory of the remaining interaction, namely quantum gravity. On the other hand, the numerous unsuccessful attempts to construct a quantum theory of gravity for more than six decades on the assumption that the present linear quantum theory is correct suggests that perhaps not only general relativity but also quantum theory should be modified in order to construct of a satisfactory quantum theory of gravity.

In section 2, I shall briefly review the measurement problem and protective observations. I shall argue, in section 3, that none of the standard interpretations of quantum theory provide a solution for the measurement problem. This suggests that a modification of quantum theory may be needed, particularly since protective observation suggests that the wave function is real and therefore the reduction of the wave packet during measurement is a real objective process. I then consider the specific suggestion along the lines mentioned above due to Roger Penrose [27, 28, 29, 30, 31]. He advocated the use of the gravitational field of the wave function to explain its reduction during measurement. Several other physicists have also argued that the phenomenon of state vector reduction is an objective, real process, and not just a change in the state of knowledge of the observer [20, 25, 17, 26, 18], an important aspect of Penrose's proposal and that of Diosi [14, 15] is that they have a quantitative prediction for the time of collapse of the wave function, which has the potential of being subject to experimental tests. Conversely, experiments could guide us in constructing a definite theory (which still does not exist) that would justify or modify this proposal.

2 Quantum measurement problem

The simplest, though dramatic, statement of the measurement problem in quantum theory is that *quantum theory does not explain the occurrence of events*. So, quantum theory does not explain the first thing we observe about the world

around us.

To see this, consider a quantum system whose state is described by a wave function ψ just before it interacts with an apparatus, which we shall treat quantum mechanically also. Suppose $\psi = \sum_i c_i \psi_i$, where ψ_i is a state of the system which after it interacts with the apparatus leaves it in the state that is described by the wave function $\psi'_i \alpha_i$, where ψ'_i represents the new state of the system and α_i the corresponding state of the apparatus. We represent this by

$$\psi_i \alpha \rightarrow \psi'_i \alpha_i \quad (1)$$

Then it follows from the linearity of quantum evolution that the interaction of ψ with the screen is represented by

$$\psi \alpha \rightarrow \sum_i c_i \psi'_i \alpha_i. \quad (2)$$

The resulting state is called an entangled state, meaning that it cannot be written as simple product of the form $\psi' \alpha'$.

For example, the quantum system may be a photon and the apparatus a photographic plate. Then ψ_i is a localized wave function of the photon which interacts with the plate to trigger a chemical reaction which results in a spot on the screen represented by α_i . But ψ produces a quantum superposition of many different spots on the screen that correspond to the different states ψ_i of the photon. Since the photon has now been absorbed, $\psi'_i \alpha_i$ in the right hand side of (1) and (2) may be replaced simply by α_i . We would not call the resulting state entangled. Nevertheless, the experiments (1) establishes a correlation between the state ψ_i and α_i . So, if we observe α_i , whether or not the quantum system is now present, we can deduce the state of the system ψ_i that would have caused this state of the apparatus. So, what is essential to measurement is this *correlation* between the system states and the corresponding apparatus states and not entanglement.

However, for a given photon in the state ψ we actually observe only one spot described, say, by α_k . This appearance of a spot may be regarded as an

approximate representation of an event, because it occurs in a fairly localized region of space-time that is defined by the small spatial region on the screen and the small interval of time during which it is formed. But (2) by itself does not explain the appearance of this ‘event’. So, we need to make an additional ‘projection postulate’

$$\sum_i c_i \psi'_i \alpha_i \rightarrow \psi'_k \alpha_k, \quad (3)$$

where $\psi'_k \alpha_k$ represent the particular event or set of events observed. The quantum measurement problem is the problem of understanding (3), which is referred to as the reduction of the wave packet or collapse of the wave function. For example, is (3) an objective dynamical process, which we may take (2) to be, or is it a subjective process we make in our minds due to the additional information we obtain from the measurement? Or what determines the *preferred states* α_i into which the reduction takes place?

So, the state vector undergoes two types of changes [32], which using the terminology of Penrose [29, 31], may be called the U and R processes. to which (1) and (2) are examples of what he calls the U process, whereas (3) is the R process. The U process is the linear unitary evolution which in the present day quantum theory is governed by Schrödinger’s equation. But what causes the measurement problem is the linearity of the U process. The unitarity is really relevant to the R process. Unitarity ensures that the sum of the probabilities of the possible outcomes in any measurement, each of which is given by an R process remains constant during the U time evolution. This of course follows from the postulate that the transition probability from the initial to the final state in the R process is the square of the modulus of the inner product between normalized state vectors representing the two states.

The process of measurement, as described above, takes place in two stages: First is the entanglement (2) and the second is the collapse (3). If we had no choice in preparing the initial state of the system then ψ is in general a superposition of the ψ_i s. Then the entanglement (2) is the inevitable consequence

of the linearity of the evolution. But if we could prepare the state then it is possible to prevent entanglement as in the case of protective observation [2]. I.e. in such an observation

$$\psi\alpha \rightarrow \psi'\alpha', \quad (4)$$

where the state represented by ψ' does not differ appreciably from the state ψ . The protection is usually an external interaction which puts ψ in an eigenstate of the Hamiltonian and the measurement process results in adiabatic evolution.

Then α' gives information about ψ ; specifically it tells us the ‘expectation value’ with respect to ψ of the observable of the system that it is coupled to an apparatus observable. By doing such experiments a large number of times it is possible to determine ψ (up to phase) even though the system is always undergoing U evolution. Consequently, the statistical interpretation of quantum mechanics is avoided during protective observations. Indeed, ψ may be determined using just one system which is subject to many experiments.

If the protection mechanism is precisely known then it would be possible to determine the ψ by means of calculation. But there is a profound difference between experimentally observing the state, which gives the manifestation of the state, and calculating it. Also, the protected state need not be in an eigenstate of the observable being measured, and yet there is no entanglement. It may appear that if the combined system evolves as (4), as in a protective measurement, then we cannot obtain new information about the system state, because if this state were previously unknown then there should be the possibility of the system being in more than one state with respect to the apparatus, i.e. there should be entanglement or correlation between system states and apparatus states. This is true if the system states already have a well defined meaning.

However, a state acquires meaning through its relation to other states. E.g. describing a state vector $|\psi\rangle$ by means of its wave function is the same as giving the inner product of $|\psi\rangle$ with all the eigenstates of the position operator. A previously proved theorem [6] states that from the ‘expectation values’ defined as

functions on the set of physical states, it is possible to construct the Hilbert space whose rays are these states. Indeed the entire machinery of quantum mechanics may be constructed from the numbers which an experimentalist obtains by protective measurements. Before the Hilbert space is reconstructed, it is not possible to calculate the wave function. However, from the information which can in principle be obtained from protective measurements, it is possible to determine the inner products between a given state vector and all other state vectors, according to the above mentioned theorem, which gives meaning to the given state vector. So if the meanings of the states are previously unknown, then in this way it is possible to obtain new information that determine the state of a system by means of protective observations, even though the evolution is according to 4. Also, this is done by using just one quantum system in the given state. No statistical interpretation of the wave function is needed. This suggests that the wave function may be real and objective.

3 Efforts to resolve the measurement problem

The well known Copenhagen interpretation attempts to deal with the measurement problem by introducing an artificial division between the quantum system being observed and the apparatus. The quantum system, which was assumed to be ‘microscopic’, is treated quantum mechanically. Its state evolves in a Hilbert space. The apparatus, assumed to be ‘macroscopic’, is treated classically. The discontinuous R process occurs when the microscopic system interacts with the macroscopic system. This is accounted for by supposing that the wave function represents only our knowledge of the state of the system, and this knowledge undergoes a discontinuous change when the measurement is made.

This is unsatisfactory because the apparatus is made up of electrons, protons, neutrons and photons, which are clearly quantum mechanical. At the time when the Copenhagen interpretation was formulated, it was not known

that a macroscopic superconductor must be treated quantum mechanically. Also, Bose-Einstein condensation, which provides another clearly macroscopic quantum system, was not experimentally realized. Today we know and possess macroscopic quantum systems. Moreover, there have been numerous quantum mechanical experiments on a macroscopic scale, using superconductors, electron, neutron and atomic interferometry. Another related problem is that the Copenhagen interpretation does not specify the line of division between the system and the apparatus. It does not give a number, specifying the complexity or mass of the system, which when exceeded would make the system macroscopic. Also, the early universe needs to be treated quantum mechanically because quantum gravitational effects were so important at that time. But nothing can be more macroscopic than the universe. And the universe is everything there is, so no line of division can be specified between it and the apparatus. Finally, protective observation, discussed in section 1, suggests that the wave function is real and objective, and is not just our knowledge of the system.

This brings us to two famous interpretations of quantum theory in which the wave function is regarded as real, consistent with the meaning of the wave function given by protective observation. One is the Everett interpretation [16] in which the wave function never collapses. But this view carries with it a huge excess intellectual baggage in the form of infinitely many worlds that coexist with the world in which we observe ourselves in. Also, it does not seem to explain the ‘preferred basis’ or the ‘interpretation basis’ in which we observe the world to have a fairly classical space- time description. The latter description is very different from the Hilbert space description which is the only reality in the Everett view. Furthermore, since the Everett view gives a deterministic description of a real state vector, the only natural way of introducing probabilities is by coarse graining. But this would not agree with the probabilities determined by the inner product in Hilbert space which is well confirmed by experiment.

The Bohm interpretation [11], tries to overcome the old problem of wave-

particle duality that asserts the simultaneous existence of both the particle and the wave. This dual ontology enables one to have the cake and eat it too. A direct experimental evidence of a particle, such as the triggering of a particle detector, or a track in a cloud chamber, etc. is explained as caused by the particle. And this is the *only* role of the particle. The motion of the particle is assumed to be guided by a ‘quantum potential’ which explains, for example, the result of an interference experiment. Without the particle the Bohm interpretation would be like the Everett interpretation in that there is no collapse of the wave function. But the particles determine which branch of the wave function we i.e. the particles constituting us) are in. So, there is no excess intellectual baggage of the many worlds as far as the particles are concerned. But there are the ‘empty waves’ of the other branches. These waves may be protectively observed and therefore may be regarded as real [10]. This has the advantage over the previous interpretations in that there is no preferred basis problem because the particles determine ‘events’, e.g. spots on the photographic plate, which give the illusion of a preferred basis in the Hilbert space.

The absence of any further role for the particle is illustrated by the fact that the particle does not react back on the wave. This violates the action-reaction principle, which may be regarded as a metaphysical objection to the Bohm interpretation [19, 10]. It is also strange that in this theory the wave function plays a dual role, namely the ontological role of guiding the particle, and the epistemological role of giving initially at least the usual prescription for the probability density of finding the particle. Also, parameters such as charge, mass, etc. which are usually associated with the particle are spreadout over the wave and not localized on the particle [12] in the Bohm picture. Finally, when one goes over to quantum field theory, the ontology undergoes a sudden change because the particle is replaced by the classical field and it is not clear what its relation is to the previous ontology.

In the Feynman path integral formalism of quantum theory, the measure-

ment problem does not seem to occur, at least not explicitly. Recently, Kaiser and Stodolsky [21] have claimed that the measurement problem does not arise in the Feynman path integral approach. In this approach one assumes ‘events’, such as the spots on a photographic plate, to be a primitive concept. Only these events are considered to be real. Given an event A caused by a system, quantum mechanics gives the probability amplitude for a subsequent event B to be caused by the same system. This is obtained by summing the probability amplitudes associated with the different paths by which the system may go from A to B . Here, like in the Copenhagen interpretation, but unlike the Everett or Bohm interpretations, the wave function is not real. It is the probability amplitude for different possible events, and is therefore a prescription for the statistical prediction of these events. It may then appear that there is no measurement problem because we can deal directly with probability amplitudes without a wave function which undergoes a mysterious collapse.

However, the measurement problem can still be formulated by means of the following three questions in the amplitude language, with the translation into the wave function language given in parentheses. 1) When do we convert probability amplitudes into probabilities? (Criterion for macroscopicity of the apparatus?) 2) Why only one of the many possible events with non zero probability amplitude is realized in a particular experiment. (Collapse problem.) 3) Why don’t we see a superposition of the states that are actually observed in experiments for which also there is a non zero probability amplitude? (The preferred basis problem.) The wave function may be regarded as the probability amplitudes to observe the particle at various points in space which then relates the above questions to the corresponding questions in the wave function language in parentheses. One cannot make the measurement problem go away by simply changing the language, which was Wigner’s answer to my question.

Although I rejected the Copenhagen interpretation as unsatisfactory, it may nevertheless be telling us something important. The apparatus being ‘classi-

cal' simply means that it should be given a space-time description. So, the preferred basis associated with the reduction mentioned in the previous section consists of states which appear 'classical', i.e. they have well defined space-time representation. The quantum system, on the other hand, has its states in the Hilbert space. But the space-time geometry is very different from the natural geometry for quantum theory which is obtained from the Hilbert space (see for e.g. [5]). The gravitational field is now incorporated into the geometry of space-time. Indeed the difficulty in constructing a quantum theory of gravity may be due to these very different geometries for space- time and Hilbert space. But the R process brings these two geometries in contact with each other because of the formation of events when the Hilbert space state vector is observed [4]. This suggests that the gravitational field may be involved in this process. If the gravitational field, which is intimately connected with space-time, causes the reduction of the wave packet, then this may explain why the states into which the collapse takes place have a well defined space-time description. Also, this argument suggests that it is not necessary to go down to the scale of Planck length for quantum gravitational effects to become important, because the above problem of relating the Hilbert space geometry to space-time geometry, which is required by the reduction of the wave packet, exists even at much bigger length scales.

4 Gravitational reduction of the wave packet

If the wave function is real, as implied by protective observation, it is likely that its collapse or reduction is also a real process. Also, as argued in the previous section, none of the interpretations of present day quantum theory advanced so far are satisfactory. We should therefore be open to the possibility of having to modify quantum theory. Several schemes have been proposed without involving the gravitational field to describe the R process, notably due to Pearle [26],

and Ghirardi, Rimini and Weber (GRW) [17]. However, as argued at the end of section 2 it is plausible that the gravitational field is involved in the reduction. Indeed several suggestions for such a reduction have been made [20, 27, 18]. But I shall consider here only a recent specific proposal by Penrose [31] which makes the same quantitative prediction for the time of collapse as Diosi [15], although Penrose's geometrical motivations are different from Diosi's.

To fix our ideas, consider the Stern-Gerlach experiment for a spin-half particle such as a neutron. It is well known that as the neutron passes through the inhomogeneous field of the Stern-Gerlach apparatus, its wave function splits into two, and when it interacts with a screen the combined wave function of the neutron and the screen also splits into two, as they undergo the linear U process of quantum mechanics. But the gravitational fields of the two states are different. So, if the gravitational field is to be treated quantum mechanically then the new state is the superposition

$$\Psi = \lambda|\psi_1 > |\alpha_1 > |\Gamma_1 > + \mu|\psi_2 > |\alpha_2 > |\Gamma_2 > = \lambda|\Psi_1 > + \mu|\Psi_2 >, \quad (5)$$

where $|\psi_1 >$ and $|\psi_2 >$ are represent the states of the neutrons, $|\alpha_1 >$ and $|\alpha_2 >$ the corresponding quantum states of the screen with the different positions of the spot where the neutron strikes, $|\Gamma_1 >$ and $|\Gamma_2 >$ are the coherent states of the gravitational field, and $|\Psi_1 >$ and $|\Psi_2 >$ represent the states of the combined system. Interesting consequences of a superposition of states of a macroscopic system of the form (5) for a cosmic string have been obtained elsewhere [7, 8].

Penrose argues [31] that in the superposition (5) there must necessarily be a 'fuzziness' in the time translation operator and a corresponding 'fuzziness' in the energy. This is important for the following reason. In a dynamical collapse model, such as Penrose's being considered here, typically there is violation of conservation of energy-momentum. In the GRW scheme, this violation occurs very rarely and so, it was claimed, that it cannot be detected in the usual experiments. But conservation laws are consequences of symmetries, which are to me the most fundamental aspects of physics. This is illustrated by the fact

that although, as mentioned above, the Hilbert space geometry and space-time geometry are very different, they have in common the action of the Poincare symmetry group on both of them, as if this symmetry group is ontologically prior to both descriptions. I expect symmetries of laws of physics and the conservation laws which they imply to be more lasting than the laws themselves. I therefore would not like even a rare violation of the conservation of energy-momentum. The ‘fuzziness’ of time translation that Penrose mentions, which may be extended also to spatial translations, may change the present laws just so as to altogether prevent the violation of energy-momentum conservation.

The uncertainty of energy associated with this ‘fuzziness’, according to Penrose, makes superpositions of the form (5) unstable. This is analogous to how the uncertainty of energy ΔE of a particle makes it unstable giving it a lifetime is of the order of $\frac{\hbar}{\Delta E}$. It is therefore reasonable to suppose that the superposed states in (5) should decay into one or other of the two states, which we observe to happen in a Stern-Gerlach experiment. The lifetime may be postulated to be

$$T = \frac{\hbar}{E}, \quad (6)$$

where E is to be determined. Penrose considers the special case of the state Ψ being an equal superposition of two states of a lump of mass, each of which produces a static gravitational field. In the Stern-Gerlach experiment considered above, this corresponds to the spin state being perpendicular to the inhomogeneous magnetic field. Define now a quantity which has dimension of energy

$$\Delta = \frac{1}{G} \int (\nabla \Phi_1 - \nabla \Phi_2)^2 d^3x, \quad (7)$$

where Φ_1 and Φ_2 are the Newtonian gravitational potentials of the two lump states, and G is Newton’s gravitational constant. Penrose [31] and Diosi [15] postulate that E is some numerical multiple of Δ .

Two questions which arise now are whether this postulate can be obtained in some natural way and how it could be generalized. I shall try to answer both questions. Note first that the classical gravitational field corresponds to the

mean value of the metric operator $\hat{g}_{\mu\nu}$ and the connection operator $\hat{\Gamma}_{\mu\nu}^\rho$. Quantum gravitational effects, however, depend on the fluctuation of the gravitational field. Consider a weak gravitational field for which the linearized approximation is appropriate. Then the gravitational fields of the superposed states may be regarded as perturbations of a background Minkowski space- time. The *fluctuation* of the connection $\Delta\Gamma$ is given by

$$\Delta\Gamma^2 = \sum \int < \Psi | (\hat{\Gamma}_{\mu\nu}^\rho - < \Psi | \hat{\Gamma}_{\mu\nu}^\rho | \Psi >)^2 | \Psi > d^3x \quad (8)$$

For (8) to be physically meaningful, it is necessary to eliminate the gauge degrees of freedom by quantizing the connection coefficients in an appropriate gauge in which these coefficients are unique. This gauge is here taken to be the gravitational analog of the electromagnetic Coulomb gauge that will be defined in the next section. Then the sum in (8) means the summing of the fluctuations of each of the operators $\hat{\Gamma}_{\mu\nu}^\rho$ defined in this gauge to represent the independent physical degrees of freedom. Then (8) may be transformed to any other gravitational gauge.

If Ψ is an eigenstate of these operators then (8) vanishes, and the geometry is essentially classical. So, we would not expect it to decay, i.e. T is infinite. It is reasonable therefore to take E to be proportional to some positive power of $\Delta\Gamma$. Since $\frac{1}{G}\Delta\Gamma^2$ has the dimension of energy, I postulate that

$$E = \frac{k}{G}\Delta\Gamma^2, \quad (9)$$

where k is some dimensionless constant to be determined by the future quantum theory of gravity.

Consider now the superposition of two gravitational fields of the form (5), where $|\Psi_1 >$ and $|\Psi_2 >$ are eigenstates of $\hat{\Gamma}$ with eigenvalues Γ_1 and Γ_2 . Then

$$\Delta\Gamma^2 = \sum \int \{ |\lambda|^2(1 - |\lambda|^2)\Gamma_1^2 + |\mu|^2(1 - |\mu|^2)\Gamma_2^2 - 2|\lambda|^2|\mu|^2\Gamma_1\Gamma_2 \} d^3x. \quad (10)$$

If the fields are Newtonian, then the only non vanishing connection coefficients (a la Newton-Cartan theory) are $\Gamma_{100}^i = \frac{\partial\Phi_1}{\partial x^i}$ and $\Gamma_{200}^i = \frac{\partial\Phi_2}{\partial x^i}$. Therefore, in the

special case considered by Penrose for which $\lambda = \mu = \frac{1}{\sqrt{2}}$, from (9) and (10),

$$E = \frac{k}{4G} \int (\nabla\Phi_1 - \nabla\Phi_2)^2 d^3x. \quad (11)$$

This E is proportional to Δ given by (7).

Hence, (9) generalizes Penrose-Diosi ansatz in three ways. We can now predict the order of magnitude of T for arbitrary coefficients λ and μ . Also, (6) is valid for superpositions of more than two lump states. Finally, we can now obtain T not only for arbitrary superpositions of static gravitational fields but also for non static gravitational fields for which there are other components of $\Gamma_{ab}{}^c$ besides $\Gamma_{00}{}^i$. For example, the above results may be applied to Leggett's proposed experiment to realize the quantum superposition of two currents in a SQUID [22, 13, 23].

The prediction (6) together with (9) for the time of reduction of the wave packet does not say how this reduction takes place. This will be considered in the last section of this paper. The question of how well the above predictions agree with experiment, for example the superposition of two currents mentioned above, will be investigated in a future paper.

5 Gravitational coulomb gauge

It has been shown that in electromagnetism the suitable gauge for studying the fluctuation of the vector potential is the Coulomb gauge [1]. I shall therefore now define the analog of the Coulomb gauge for the gravitational field and require that (8) is defined in this gauge.

In the weak field limit, we may write $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$, where $\gamma_{\mu\nu} \ll 1$. Then I shall define the gravitational Coulomb gauge by

$$\sum_{j=1}^3 \gamma^{\mu j}{}_{,j} = 0 \quad (12)$$

The linearized Einstein field equations are [24]

$$\gamma_{\mu\alpha,\nu}{}^\alpha + \gamma_{\nu\alpha,\mu}{}^\alpha - \gamma_{\mu\nu,\alpha}{}^\alpha - \gamma_{,\mu\nu} - \eta_{\mu\nu}(\gamma_{\alpha\beta,}{}^{\alpha\beta} - \gamma_{,\beta}{}^\beta) = 16\pi T_{\mu\nu} \quad (13)$$

where $,\mu$ represents partial derivative with respect to x^μ , $\gamma = \gamma_\alpha{}^\alpha$ and repeated greek indices are summed over 0, 1, 2, 3. The metric has signature $(-+++)$.

On imposing the gauge condition (12), (13) reads

$$\gamma_{\mu 0,\nu}{}^0 + \gamma_{\nu 0,\mu}{}^0 - \gamma_{\mu\nu,\alpha}{}^\alpha - \gamma_{,\mu\nu} - \eta_{\mu\nu}(\gamma_{00,00} - \gamma_{,\beta}{}^\beta) = 16\pi T_{\mu\nu} \quad (14)$$

The $(\mu, \nu) = (0, 0), (i, 0)$ and (i, j) components of (14) are respectively

$$-\sum_{k=1}^3 \sum_{m=1}^3 \gamma_{kk,mm} = 16\pi T_{00}, \quad (15)$$

$$-\sum_{j=1}^3 (\gamma_{i0,jj} + \gamma_{jj,i0}) = 16\pi T_{i0} \quad (16)$$

and

$$\gamma_{i0,j}{}^0 + \gamma_{j0,i}{}^0 - \gamma_{ij,\alpha}{}^\alpha + \gamma_{00,ij} - \sum_{k=1}^3 h_{kk,ij} - \delta_{ij} \sum_{k=1}^3 (\gamma_{00,kk} - \gamma_{kk,\beta}{}^\beta) = 16\pi T_{ij} \quad (17)$$

I assume now that each $\gamma_{\mu\nu}$ falls off sufficiently rapidly at infinity so that the Poisson's (15) determines $\sum_{k=1}^3 \gamma_{kk}$ in terms of T_{00} uniquely. Substituting this into (16), γ_{i0} is determined uniquely in terms of T_{00} and T_{i0} . Now take trace of (17) by summing over $i = j = 1, 2, 3$. Substitute for $\sum_{k=1}^3 \gamma_{kk}$ and γ_{i0} the values we have just found. Then γ_{00} satisfies Poisson equation with the source being a function of $T_{\mu\nu}$. Hence, γ_{00} is determined uniquely.

It is easy to show that the remaining components γ_{ij} ($i, j = 1, 2, 3$) of $\gamma_{\mu\nu}$ are determined uniquely by (12). To see this, do an infinitesimal gauge transformation $x^{\mu'} = x^\mu - \xi^\mu$. Then the corresponding transformation of $g_{\mu\nu}$ is equivalent to the transformation

$$\gamma'_{\mu\nu} = \gamma_{\mu\nu} + \xi_{\mu,\nu} + \xi_{\nu,\mu}. \quad (18)$$

Considering $(\mu, \nu) = (i, j)$ and requiring (12) in the new gauge also,

$$\sum_{j=1}^3 (\xi_{i,jj} + \xi_{j,ij}) = 0. \quad (19)$$

In momentum space¹ (19) reads

$$\mathbf{k}^2 \tilde{\xi}^i + (\mathbf{k} \cdot \tilde{\xi}) k^i = 0, \quad (20)$$

where $\tilde{\xi}^i$ is the Fourier transform of ξ^i . Therefore, $\tilde{\xi}^i$ is proportional to k^i and so we can write $\tilde{\xi}^i = \alpha(\mathbf{k}) k^i$. Substituting into (20), either $\alpha = 0$ or $\mathbf{k}^2 = 0$, which means that ξ^i has no spatial dependence. It follows that, in either case, from (18), $\gamma'_{ij} = \gamma_{ij}$. But it was shown earlier that $\gamma_{\mu 0}, \mu = 0, 1, 2, 3$ are uniquely determined in this gauge. Hence, all $\gamma_{\mu\nu}$ are uniquely determined in the gauge (12).

An interesting aspect of this gauge is that in the absence of matter, the above results imply

$$\gamma_{\mu 0} = 0, \mu = 0, 1, 2, 3, \sum_{k=1}^3 \gamma_{kk} = 0, \sum_{j=1}^3 \gamma_{ij,j} = 0. \quad (21)$$

I.e. the above gravitational coulomb gauge reduces to the usual transverse traceless gauge in the absence of matter, which is the physical gauge for gravitational radiation.

6 Laws, symmetry and the measurement process

It was mentioned at the beginning of section 2 that quantum theory does not explain the occurrence of events. ‘Explain’ here implicitly assumes having a causal law that describes the formation of events. But the very notion of law is strange in that it carries with it a necessity which is not logical or mathematical necessity. This is because a law must be refutable, whereas a logical or mathematical necessity is tautological and therefore cannot be refuted. It is therefore reasonable to consider the consequences of there not being any laws of necessity.

¹This argument was made in collaboration with Joseph Samuel.

A law of necessity, or simply a law, may be defined as the ability to describe the initial state of a physical system in such a way that the final state may be predicted uniquely from this initial state using the nature of the system and its interaction with its environment. The absence of laws then implies that identical physical systems may start from the same initial state and end up in different final states. This statement is consistent with our observation of quantum phenomena. However, as mentioned in section 3, attempts were made to violate this statement and make quantum theory conform to the paradigm that all phenomena occur according to laws.

By means of protective measurements [2], which we can do in principle, any state in the Hilbert space can be observed for a single system. I shall therefore allow any state in the Hilbert space to be the initial or final state of a system. However, we observe macroscopic systems in states in which the wave packets of its constituents are localized. Hence, even if it is initially in a state whose wave function is spread out then it could end up in a state which is sufficiently well localized. According to the hypothesis advanced here, this process is not described by a deterministic law. Nevertheless, the prediction (6) together with (9) will apply to the time taken for the initial state to become the final state.

But in order to give up laws, it is necessary to provide an alternative explanation for the regularities in the phenomena which we observe. E.g. why do planets have seemingly precise orbits? Or why are there precise experimentally well confirmed probabilities for the possible final states of a given initial state of a quantum system? I believe that regularities such as these may be explained by symmetries.

As for the first question, the seemingly precise motions in classical physics must be obtained as appropriate limiting cases of the quantum motion, which so far has been described by the motion of a wave function. The external field modifies this motion by means of phase shifts in interfering secondary wavelets in the propagation of this wave via Huygens principle. But it was shown that

the phase shifts due to gravity and gauge fields are caused by elements of the Poincare group and the corresponding gauge group, respectively [3, 8]. These phase shifts of course were obtained from laws which have these symmetries. But once having obtained them we could give up the laws and keep only the symmetries.

As for the second question, consider the tossing of a coin. The equal probabilities for ‘heads’ and ‘tails’ is due to the symmetry between them. The probabilities are independent of the particular law governing the motion of the coin, so long as this law respects this symmetry. This suggests that the probabilities may be independent of the existence of laws, and governed only by the symmetries. It may be possible to deduce the quantum mechanical probabilities by symmetry considerations alone.

If we give up the laws then the problem of energy-momentum non conservation in the dynamical collapse models, which was mentioned in section 3, disappears. We simply accept the transition of a quantum state from a given initial state to a final state, without a law governing this process, in such a way that energy-momentum and all other conserved quantities corresponding to the known symmetries are conserved for the combined system consisting of each observed system and its environment. In quantum theory there is a direct connection between the symmetries and conserved quantities due to the symmetries that act on the Hilbert space being generated by the conserved quantities.

Another problem with dynamical collapse, if we work within the paradigm of laws, is that if a charged particle wave function collapses we would expect it to radiate, which has not been observed. But if we give up laws and work in a new paradigm in which symmetries are the fundamental concepts then again we can simply accept the transition of this wave function from the initial to the final state without requiring a law to account for the motion in between. This transition then could happen without any radiation.

But a great deal of work remains to show that all the regularities that we ob-

serve in nature could be obtained from symmetries and logical or mathematical self consistency. This will be explored in another paper [9].

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